**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Sol:-**

**We have a normal distribution with = 45 and = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).**

**Pr(X > 50) = 1 - Pr(X ≤ 50).**

**Z = (X - )/ = (X - 45)/8.0**

**Thus the question can be answered by using the normal table to find**

**Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%**

**Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676**

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**Sol:-**

**False.** If more employees are older than 44, this will shift the *μ* towards 44 with considering sd. Which is not possible as *μ* is given 38 with sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Sol:-**

**True.**

Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X -*μ* )/ *σ* = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Sol:-**

**As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,σ12+σ22). And 2X1 will just scale the normal distribution by 2 times**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**SOL:-**

**x=σ[z] + µ**

**Thus "a" = 0.5th percentile for X = 20\*[-2.57] + 100 = 48.5**

**and "b" = 99.5th percentile for X = 20\*[+2.57] + 100 = 151.4**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**SOL:-**

**norm.ppf(0.025,45\*5,3)**

**219.12**

**norm.ppf(0.975,45\*5,3)**

**230.87**

**norm.ppf(0.025,45\*7,3)**

**309.12**

**norm.ppf(0.975,45\*7,3)**

**320.87**

**So the Rupee range with 95% probability for the annual profit of the company is given by,**

**=Profit1 + Profit2**

**= [219.12, 230.87] + [309.12, 320.87]**

**= [528.24, 551.74]**

**So Profit of the company in Rupees(in Million) is between range [528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**SOL:-**

**norm.ppf(0.05,45\*7,3)**

**310.065**

**norm.ppf(0.05,45\*5,3)**

**220.065**

**5th percentile of profit = 310.0654+ 220.0654 = 530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**SOL:-**

**Division 2 with distribution N(7, 42)**